Cooper Pair Formation in U(1) Gauge Theory of High Temperature Superconductivity

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We study the two-dimensional spin-charge separated Ginzburg-Landau theory containing U(1) gauge interactions as a semi-phenomenological model describing fluctuating condensates in high temperature superconductivity. Transforming the original GL action, we abstract the effective action of Cooper pair. Especially, we clarify how Cooper pair correlation evolves in the normal state from the point of view of spin-charge separation. Furthermore, we point out how Cooper pair couples to gauge field in a gauge-invariant way, stressing the insensitivity of Cooper pair to infrared gauge field fluctuation.

KEYWORDS: high temperature superconductivity, RVB theory, gauge theory, Ginzburg-Landau theory, superconducting fluctuation

The mechanism of high temperature superconductivity remains still controversial after all the intensive efforts over a decade. Among them, two-dimensional t-J model has been studied as one of the feasible microscopic models describing some anomalous properties in the normal state. In the light of possible spin-charge separation in the elementary excitations, the phase diagram based on slave boson mean field approximation (MFA) [1] shows fair correspondences with some experimental facts and seems to indicate the origin of pseudogap. On the other hand, it has also been pointed out[2] that only superconducting transition will remain if one takes the gauge field coupling into account as the fluctuations around MFA, while that of spinon and holon will vanish as artifacts accompanied with MFA and turn into just crossovers. One of the functions of gauge field is to connect spin- and charge-subsystem by eliminating the redundant degrees of freedom in the slave boson representation. For instance, it appears in the electromagnetic responses of total system as Ioffe-Larkin composition rule.[3] In addition, the gauge field is considered to cause T-linear resistivity in the normal state by scattering charged particles(holons).[4] Thus gauge field may play essential roles in doped Mott insulators as a reflection of strong correlation among electrons in two dimensions.

When we consider the properties involving superconducting order separately from the elementary excitations in the normal state, what we need is a knowledge about the correlations of Cooper pair as a composite of spin and charge, not that of each constituent. In this Letter, we target how Cooper pair correlation evolves in spin-charge separated description. Starting from spin-charge separated Ginzburg-Landau (GL) theory containing U(1) gauge field, we abstract the Cooper pair and its effective theory. We shall also see how Cooper pair 'feels' gauge field and is affected by its fluctuation, in comparison with the case of the separate constituents. In the following, we set $\hbar = k_{\mathrm{B}} = c = 1$.

We consider the circumstance where both pairing order parameters of spin and charge degrees of freedom are fluctuating with zero mean value, which is a possible model describing the fluctuations of condensates in the context of Resonating Valence Bond (RVB) theory. For simplicity, we ignore the imaginary time dependence of the action. Our starting point is two-dimensional, two-component Ginzburg-Landau action:

$$\begin{aligned} &S[\bar{s}, s, \bar{h}, h, \boldsymbol{a}] \\ &= \frac{1}{T}\int \mathrm{d}\boldsymbol{r}\,[\ \alpha_s|s(\boldsymbol{r})|^2 + c_s|(-\mathrm{i}\nabla + \boldsymbol{a}(\boldsymbol{r}))s(\boldsymbol{r})|^2 \\ &+\alpha_h|h(\boldsymbol{r})|^2 + c_h|(-\mathrm{i}\nabla - \boldsymbol{a}(\boldsymbol{r}))h(\boldsymbol{r})|^2 \\ &+\frac{u_s}{2}|s(\boldsymbol{r})|^4 + \frac{u_h}{2}|h(\boldsymbol{r})|^4 + u_c|s(\boldsymbol{r})|^2|h(\boldsymbol{r})|^2\ ], \end{aligned} \quad (1)$$

thereby the partition function expressed as

$$Z = \int D\bar{s}DsD\bar{h}DhDa\exp[-S[\bar{s}, s, \bar{h}, h, \boldsymbol{a}]]. \quad (2)$$

We take Coulomb gauge $\nabla\cdot\boldsymbol{a}(\boldsymbol{r}) = 0$. Similar model has been adopted in several papers to investigate the properties concerning vortices in spin-charge separated systems.[2,5] Complex scalar fields $s(\boldsymbol{r})$ and $h(\boldsymbol{r})$ represent the pairs of spinon and antiholon, respectively. Although either of them should couple to the electromagnetic field, we set it aside for the present. This action has a symmetry under the local U(1) gauge transformation

$$\begin{aligned} &s(\boldsymbol{r}) \to s(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r})},\ h(\boldsymbol{r}) \to h(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}\theta(\boldsymbol{r})}, \\ &\boldsymbol{a}(\boldsymbol{r}) \to \boldsymbol{a}(\boldsymbol{r}) - \nabla\theta(\boldsymbol{r}). \end{aligned} \quad (3)$$

Each component couples to common U(1) gauge field $\boldsymbol{a}(\boldsymbol{r})$. Although $\boldsymbol{a}(\boldsymbol{r})$ should have its own kinetic term stemming from polarizations of the normal components,[2] we omitted it to concentrate our attention upon the dominant couplings to the fluctuations of the conden-

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sates. Due to such implicit kinetic term, we assume that the gauge coupling is already in the weak-coupling region so that the perturbative treatment is justified. The stability of the system requires the condition $u_s u_h - u_c^2 > 0$. The last term in the action(1) is related to the correlation of fluctuations between the components and therefore negative u_c can be considered to accelerate the formation of Cooper pairs. In the following, we limit our considerations to the cases $u_c < 0$.

Before analyzing the properties involving Cooper pair, let us give a brief overview of some aspects of the model(1). For a while, we can consider the situation as if each component is fluctuating independently, deferring the considerations of the gauge coupling. At certain temperature around $\alpha_i = 0$ $(i = s, h)$, each component is expected to go through a crossover from Gaussian to XY-like fluctuation, establishing a finite amplitude without spontaneous symmetry breakings. This amplitude is considered to cause a gap in such elementary excitation as spinon in the context of slave boson mean field theory.[1] We can also consider Kosterlitz-Thouless (KT) transition[6] in each subsystem. Due to finite gauge coupling, however, the independent two-component phase fluctuation proves to be just a fictitious vision which will never occur in the real system. Especially in the strong-coupling limit, it has been shown in several papers[5,7] that after integrating out the gauge field first there will finally remain three physical degrees of freedom: amplitudes $|s|, |h|$ and phase sum $\arg(sh)$. Namely, it is (quasi-)long-range order of the phase of Cooper pair that survives even in the strong-coupling limit. That is why we need to construct the effective theory of Cooper pair.

To see the kinetics of Cooper pair in the original action(1), we introduce auxiliary field $\Delta(\boldsymbol{r}) \sim u_c s(\boldsymbol{r}) h(\boldsymbol{r})$:

$$\begin{aligned}&\exp\left[-\frac{1}{T}\int \mathrm{d}\boldsymbol{r} u_c |s(\boldsymbol{r})|^2 |h(\boldsymbol{r})|^2\right]\\ &= \int D\bar{\Delta} D\Delta \exp\Big[-\frac{1}{T}\int \mathrm{d}\boldsymbol{r}[u_c^{-1}\bar{\Delta}(\boldsymbol{r})\Delta(\boldsymbol{r})\\ &\quad -\bar{\Delta}(\boldsymbol{r})s(\boldsymbol{r})h(\boldsymbol{r}) - \Delta(\boldsymbol{r})\bar{s}(\boldsymbol{r})\bar{h}(\boldsymbol{r})]\Big]. \qquad (4)\end{aligned}$$

Then we have an action $S[\bar{s}, s, \bar{h}, h, \bar{\Delta}, \Delta, \boldsymbol{a}]$. The gauge symmetry of $\bar{\Delta}(\boldsymbol{r})s(\boldsymbol{r})h(\boldsymbol{r})$ and its complex conjugate implies that Cooper pair $\Delta(\boldsymbol{r})$ is not transformed by (3); $\Delta(\boldsymbol{r})$ is 'neutral' and not coupled to gauge field $\boldsymbol{a}(\boldsymbol{r})$ through covariant derivatives. Such coupling would be possible to the electromagnetic field. We shall later see that Cooper pair couples to gauge field $\boldsymbol{a}(\boldsymbol{r})$ in an alternative way. If we integrate out $\bar{s}(\boldsymbol{r}), s(\boldsymbol{r})$ and $\bar{h}(\boldsymbol{r}), h(\boldsymbol{r})$, we obtain the effective action with respect to Cooper pair and gauge field as $S[\bar{\Delta}, \Delta, \boldsymbol{a}]$:

$$\begin{aligned}&S[\bar{\Delta}, \Delta, \boldsymbol{a}]\\ &= \frac{1}{T}\int \mathrm{d}\boldsymbol{r}\ [\ \alpha|\Delta(\boldsymbol{r})|^2 + c|\nabla\Delta(\boldsymbol{r})|^2 + \frac{u}{2}|\Delta(\boldsymbol{r})|^4 + \cdots\\ &\quad +\chi(\nabla\times\boldsymbol{a}(\boldsymbol{r}))^2 + \cdots\\ &\quad +v|\Delta(\boldsymbol{r})|^2(\nabla\times\boldsymbol{a}(\boldsymbol{r}))^2 + \cdots\ ]\,. \qquad (5)\end{aligned}$$

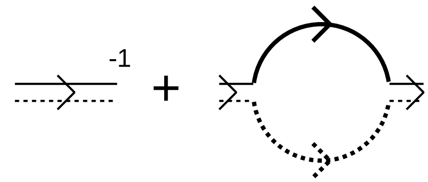

Fig. 1. The formation of Cooper pair propagator. Double lines represent Cooper pair. It acquires its own kinetics from spinon pair(solid lines) and antiholon pair(dashed lines). Thick lines mean the renormalized propagators by each quartic term(see Fig.2).

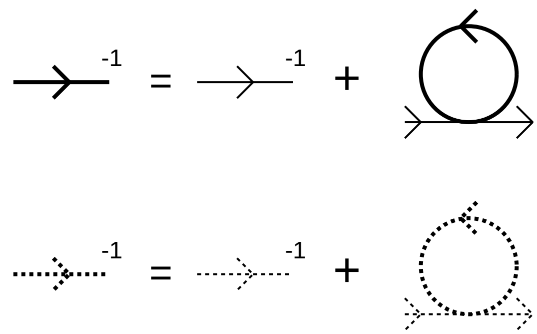

Fig. 2. Renormalizations of propagators of spinon pair and antiholon pair by each quartic term.

First we analyze the mechanism of the Cooper pair formation from the viewpoint of its constituents. We can derive the Cooper pair propagator from the processes shown in Fig.1. It is calculated by means of the propagators of spinon pair and antiholon pair as if each component has independent kinetics described by GL theory, deferring the integrating out gauge field; it is sufficient to renormalize the propagator by each quartic term. Thus the coefficients α and c are calculated as

$$\alpha = -u_c^{-1} - \frac{T}{4\pi}\frac{1}{\tilde{c}_s\tilde{c}_h}\frac{\ln(\tilde{\alpha}_s/\tilde{c}_s) - \ln(\tilde{\alpha}_h/\tilde{c}_h)}{\tilde{\alpha}_s/\tilde{c}_s - \tilde{\alpha}_h/\tilde{c}_h}, \qquad (6)$$

$$\begin{aligned}c = \frac{T}{2\pi}\frac{1}{\tilde{c}_s\tilde{c}_h}\Bigg[&\frac{(\tilde{\alpha}_s/\tilde{c}_s + \tilde{\alpha}_h/\tilde{c}_h)(\ln(\tilde{\alpha}_s/\tilde{c}_s) - \ln(\tilde{\alpha}_h/\tilde{c}_h))}{2(\tilde{\alpha}_s/\tilde{c}_s - \tilde{\alpha}_h/\tilde{c}_h)^3}\\ &-\frac{1}{(\tilde{\alpha}_s/\tilde{c}_s - \tilde{\alpha}_h/\tilde{c}_h)^2}\Bigg], \qquad (7)\end{aligned}$$

$\tilde{\alpha}_i$ and $\tilde{c}_i$ $(i = s, h)$ meaning the renormalized coefficients by each quartic term. Here we adopt the approximation as shown in Fig.2. These diagrams mean that we take only the renormalization of α_i by u_i into account and neglect any other renormalizations such as the renormalization of u_i by u_i. We shall later see that the renormalization of α_i is essential for understanding the nature of Cooper pair fluctuation in the 'spin gap' phase in the slave boson MFA.[1] In this approximation we set $\tilde{c}_i = c_i$. The renormalized coefficient $\tilde{\alpha}_i$ is determined by α_i:

$$\tilde{\alpha}_i = \alpha_i + \frac{Tu_i}{2\pi c_s}\ln\frac{c_i\Lambda^2 + \tilde{\alpha}_i}{\tilde{\alpha}_i}, \qquad (8)$$

where ultraviolet cutoff Λ is introduced in momentum space. $\tilde{\alpha}_i$ has asymptotic forms

$$\tilde{\alpha}_i \simeq \alpha_i \ (\alpha_i \to \infty), \tag{9}$$

$$\tilde{\alpha}_i \simeq \exp(2\pi c_i \alpha_i / T u_i) \ (\alpha_i \to -\infty). \tag{10}$$

We give plots of the renormalized coefficients $\tilde{\alpha}_i$ as functions of the bare coefficients α_i in Fig.3. Here we set $\Lambda = 1$, $c_s = c_h = 1$, $u_s/2\pi = 0.02$, $u_h/2\pi = 0.2$, which means the fluctuation of holon pair is much larger than that of spinon pair. Around $\alpha_i = 0$, the crossovers from Gaussian to XY-like fluctuations are expected in the virtual subsystems described by GL theory.

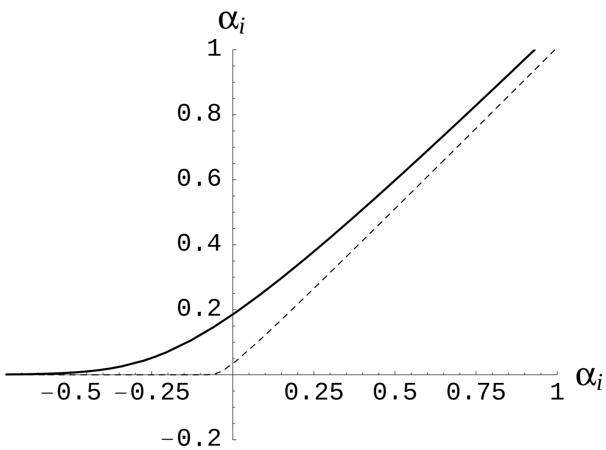

Fig. 3. The relation between bare coefficient α_i and renormalized one $\tilde{\alpha}_i$. In this case holon pair(solid line) has much larger fluctuation than spinon pair(dashed line).

Now we can see the evolution of the superconducting correlation through GL coefficient α of Cooper pair. We give a contour plot of α in Fig.4 based on eq.(6). Here we set $\alpha_i = \alpha'_i(T - T_i^0)$ $(i = s, h)$, $\alpha'_s = \alpha'_h = 1$, $T_s^0 = 1 - x$, $T_h^0 = x$, where x can be regarded as a parameter corresponding to doping rate. Bare transition temperatures T_i^0 $(i = s, h)$ and *fictitious* KT transition temperatures T_i $(i = s, h)$ are also plotted. T_i are estimated as

$$T_i = \frac{T_i^0}{1 + u_i/\pi\alpha'_i c_i}, \tag{11}$$

following Halperin and Nelson.[8] With the temperature decreasing, α decreases monotonically. We expect the superconducting (KT-)transition temperature T_c is just below such temperature as $\alpha = T - T_c^0 = 0$. Since u_c accelerates Cooper pair formation, the smaller is $|u_c|$, the lower are T_c^0 and T_c. Combining Figs.3 and 4, one can see that the sharpness of the crossover is reflected in the sharpness of the drop in α. As a result, in the underdoped region, α has two sharp drops, and in return, the superconducting fluctuation is larger than the overdoped region. Even if T_i and T_i^0 lose their meanings as characteristic temperatures for the orderings of spinon pair or holon pair due to finite gauge couplings, they leave the traces on the evolution of Cooper pair correlation, as explained in the following.

So far we have deferred the considerations of gauge

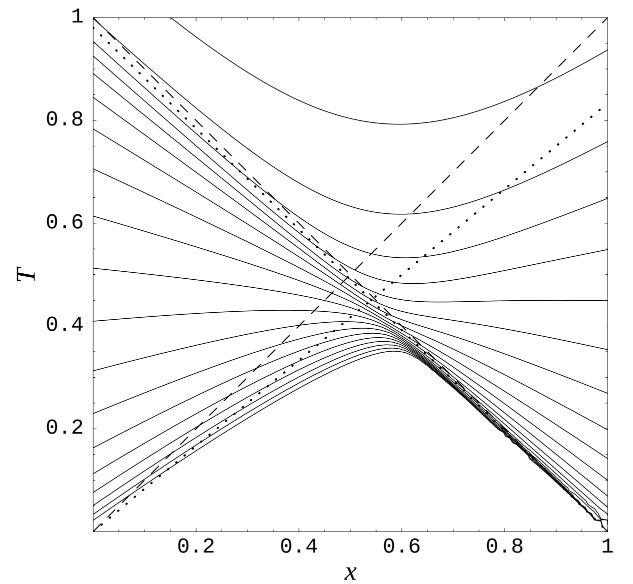

Fig. 4. A contour plot of GL coefficient α of Cooper pair as a function of temperature and doping rate. To see the wide-ranging variation, $\log(\alpha + u_c^{-1})$ is plotted instead of α itself. Bare transition temperatures $T_s^0 = 1 - x$, $T_h^0 = x$ (dashed lines) and *fictitious* KT transition temperatures $T_i = T_i^0(1 + u_i/\pi\alpha'_i c_i)^{-1}$ $(i = s, h)$ (dotted lines) are also plotted.

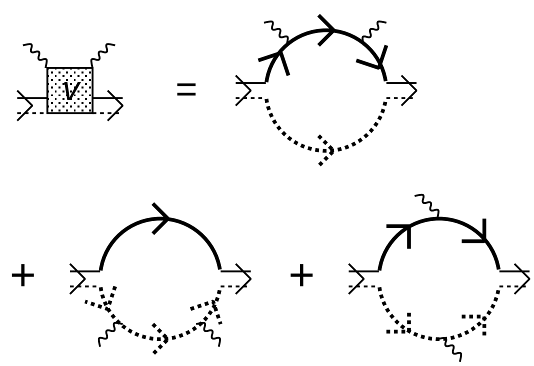

Fig. 5. The lowest order coupling between Cooper pair and gauge field(wavy lines).

field on the ground that its effective coupling is comparatively weak. In fact Cooper pair has a coupling to gauge field. Such processes in the lowest order are depicted in Fig.5. One can confirm the cancellation of minimal couplings between Cooper pair and gauge field by letting momenta of gauge field zero. Such cancellation in each order makes the destruction of ordering in s and h by infrared gauge field fluctuation invisible to Cooper pair. Giving finite momentum to gauge field and expanding with respect to it, we evaluate the vertex $v|\Delta|^2(\nabla \times \boldsymbol{a}(\boldsymbol{r}))^2$ as

$$v = \frac{T}{24\pi} \frac{c_s/\tilde{\alpha}_s + c_h/\tilde{\alpha}_h}{\tilde{\alpha}_s \tilde{\alpha}_h}. \tag{12}$$

This vertex should renormalize the Cooper pair propagator when gauge field is integrated out. However, because the gauge field never appears in the effective action $S[\bar{\Delta}, \Delta, \boldsymbol{a}]$ without accompanying differential like

$\nabla \times \boldsymbol{a}(\boldsymbol{r})$, Cooper pair is considered to be much less affected by infrared gauge field fluctuations, in striking contrast to spinon pair or holon pair.

In concluding, we have abstracted the kinetics of Cooper pair under the superconducting fluctuation from the spin-charge separated Ginzburg-Landau theory with U(1) gauge field. Our approach, which starts from the Gaussian fluctuations of two components in high temperature and renormalizes it, is complementary to that of Rodriguez,[7] where the amplitudes of the order parameters are fixed.

We first derived GL coefficients of Cooper pair in general forms from the kinetics of its components. Next we evaluated the renormalized coefficients $\tilde{\alpha}_i$, which proved to be essential for treating the region where at least one of the components has XY-like fluctuation below T_i^0. We assumed that the holon pair had much larger fluctuation than spinon pair; it is the factor $u_i/\alpha_i' c_i$ that determines the magnitude of the fluctuation of each order parameter. In this case the sharp drop in α separates into two pieces in underdoped region, located around T_s and T_h, while it concentrates around T_s in overdoped region. That leads to the enhancement of superconducting fluctuation in underdoped region.

In terms of gauge field, we implicitly assumed that its coupling is in the weak-coupling region on the ground that gauge field should already have its own kinetic term due to the polarization of the normal component. Because of the absence of the 'charge' of Cooper pair, it does not couple to gauge field in a minimal way. Instead we have pointed out the existence of the alternative couplings. We expect that the order of Cooper pair will not be destroyed critically by gauge field fluctuation because gauge field always appears as its derivative, unlike spinon pair or holon pair which directly connects to gauge field through minimal coupling.

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